Widening as Abstract Domain

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$x \nabla y$

Widening in Abstract Interpretation

Static program analysis:

- use abstract domains to represent program states
- execute abstract semantics of program statements
- compute a fixpoint that over-approximates all possible program behaviors

```
int x = y = 0;
while (x < 6) {
p_0:
  x = x + 1;
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}
p_1:
```
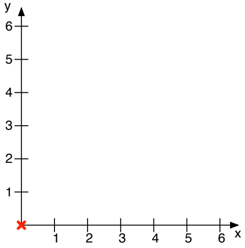

State at p_0:



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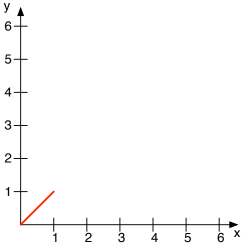

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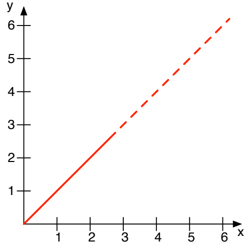
State at `p_0`: (widened)



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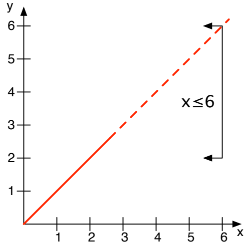
State at `p_0`: (narrowed)



Widening in Abstract Interpretation

Idea of widening:

- some domains have infinite ascending chains:
  $[0,0] \quad [0,1] \quad [0,2] \quad \ldots$
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Definition:
Given a domain $\mathcal{D}$, define $\nabla : \mathcal{D} \times \mathcal{D} \to \mathcal{D}$ such that $\forall x, y \in \mathcal{D}$:

$$x \sqsubseteq x \nabla y \quad \text{and} \quad y \sqsubseteq x \nabla y$$

and for all increasing chains $x_0 \sqsubseteq x_1 \sqsubseteq \ldots$ the increasing chain $y_0 = x_0, \ldots y_{i+1} = y_i \nabla x_{i+1}$ is eventually stable.

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- widening *seems* to require a modified fixpoint computation
- cannot easily adapt widening strategies

Properties of Narrowing

Narrowing is often required after widening:

- widening introduces imprecision by overshooting the fixpoint
- *narrowing* can sometimes recover precision
- here: 2nd iter. $p_0 : x = y, x \in [0, \infty]; p_1 : x = y, x \in [6, \infty]$
  3st iter. $p_0 : x = y, x \in [0, 5]; p_1 : x = y, x \in [6, 6]$

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int x = y = 0;
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Problems:

```
int x = y = 0;
while (x < 6) {
p_0:
  x = x + 1;
  y = y + 1;
}
p_1:
```

- need to refine states on all exit points of the loop
- what if the program contains `goto p_1` ?
- alternative: avoid propagating to p_1 until loop is stable
- complicates fixpoint engine and state management

Widening on Low Level Code

We analyze machine code:

- Control-Flow Graph (CFG) is reconstructed on-the-fly
- $\rightarrow$ loops entries and exits not known up front
- possibly irreducible CFGs: no best set of widening points
- $\rightarrow$ need a very robust widening
- $\rightarrow$ we need to try other heuristics
- $\rightarrow$ avoid narrowing altogether

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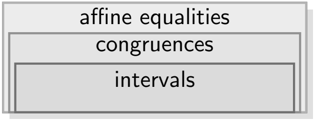
Our goal: keep fixpoint engine, implement widenings as plug-ins

Co-fibered Abstract Domains

A co-fibered domain $\langle \mathcal{D} \rhd \mathcal{C}, \sqsubseteq_{\mathcal{D}\rhd\mathcal{C}}, \sqcup_{\mathcal{D}\rhd\mathcal{C}}, \sqcap_{\mathcal{D}\rhd\mathcal{C}} \rangle$ tracks values of the form $\langle d, c \rangle \in \mathcal{D} \rhd \mathcal{C}$ where:

- d is the internal information tracked by the domain
- c is the child domain
- all operations are defined on $\langle d, c \rangle$
- $\rightarrow$ can execute multiple operations on the child or none at all
- can translate an operation on $\langle d, c \rangle$ into a different operation on the child
- example: congruence domain stores $x/4$ in child if x is multiple of 4

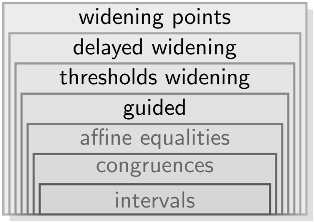


Widening as Co-fibered Domains

Idea:
implement widening + heuristics as co-fibered abstract domains.

Namely:

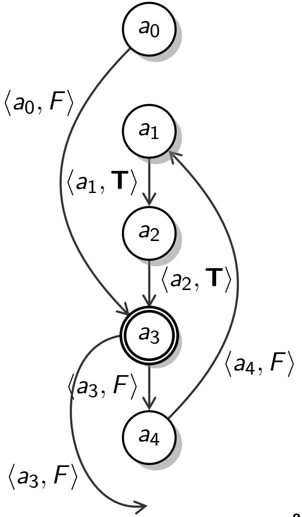
- $\mathcal{W}$: domain inferring widening points
- $\mathcal{D}$: delay domain
- $\mathcal{T}$: widening thresholds domain
- $\mathcal{P}$: guided static analysis domain



Finding Widening Points

Define domain $\mathcal{W} \rhd \mathcal{C}$ where $\mathcal{W} = Lab \times \{T, F\}$ that applies widening instead of join on child $\mathcal{C}$.

- $l \in Lab$ is a program point and $f \in \{T, F\}$ is a Boolean flag
- for termination at least one widening point in each loop is needed
- use total order on the program points (instruction addresses) to detect back-edges
- simple heuristic: any back-edges is considered an edge to a loop head
- l is smallest previous edge, f is set if back-edge has been seen



Tracking Widening Thresholds

Define $\mathcal{T} \triangleright \mathcal{C}$ where $\mathcal{T} : \mathit{Lab} \times \mathit{Pred} \times \wp(\mathit{Lab})$ that applies thresholds after widening to refine the state.

```
1 int x = y = 0;
2 while (x < 6) {
3 p_0:
4     x = x + 1;
5     y = y + 1;
6 }
7 p_1:
```

- $l \in \mathit{Lab}$ is the origin of test $p \in \mathit{Pred}$ and $a \in \wp(\mathit{Lab})$ tracks application sites of p
- track redundant tests as *thresholds*
- thresholds are invariants for the current state (applying the test does not change the state)
- here $x < 6$ is a threshold at line 3
- thresholds are transformed by assignments, so that they stay invariant
- use thresholds after widening to immediately restrict the widened state

Tracking Widening Thresholds

```
1 int x = y = 0;
2 while (x < 6) {
3 p_0:
4   x = x + 1;
5   y = y + 1;
6 }
7 p_1:
```

- collect threshold from redundant test
  3: $t = \langle 2 \times (x < 6) \times \{\} \rangle$
- transform thresholds with instructions
  4: $t = \langle 2 \times (x < 6) \times \{\} \rangle$
  5: $t = \langle 2 \times (x < 7) \times \{\} \rangle$
- apply thresholds only once per widening point (termination)
  2': $t = \langle 2 \times (x < 7) \times \{\} \rangle$
  3': $t = \langle 2 \times (x < 7) \times \{2\} \rangle$
  $\rightarrow p_0 : x = y, x \in [0, 5]; p_1 : x = y, x \in [6, 6]$
- when seeing a threshold again, keep the transformed one (termination)
- use only the "smallest" thresholds to restrict widening (retain others)

No Widening after Constant Assignments

Define $\mathcal{D} \triangleright \mathcal{C}$ where $\mathcal{D} : \wp(\mathit{Lab})$ is a set of program points with constant assignments.

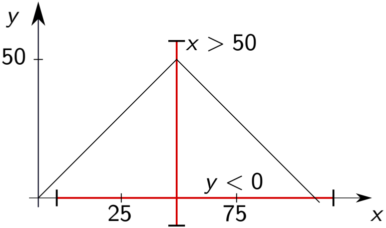
```
int x = 0;
int y = 0;
while (x < 100){
  if (x > 5) {
    y = 1;
  }
  x = x + 4;
}
```

- problem: widening of y yields $[0,0]\nabla[1,1] = [0,\infty]$
- common approach is to delay widening for the first n loop iterations (here: $n = 2$)
- slows down fixpoint computation unnecessarily if not needed
- better: do not widen if we have seen a new constant assignment
- we track program locations with constant assignments
- when widening $\mathcal{D} \triangleright \mathcal{C}$, compute a join on $\mathcal{C}$ if there are new constant assignments

Guided Static Analysis as Abstract Domain

Define $\mathcal{P} \triangleright \mathcal{C}$ where $\mathcal{P} : \mathcal{C} \times (Pred \times \mathcal{P})^* \times \wp(Pred)$.

```
int x = 0;
int y = 0;
while (true) {
  if (x <= 50){
    y++;
  } else {
    y--;
  }
  if (y<0)
    break;
  x++;
}
```



- numeric domains usually are convex approximations
- $\rightarrow$ precision loss when joining different states
- idea is to separate the states that belong to different *phases* of a loop to avoid convex approximation of widened states

Conclusion

- widening/narrowing is a challenge to implement for binary analysis
- combine with interesting widening heuristics in the literature!
- co-fibered domains allow the modular combination of different strategies
- no adjustment to the fixpoint and state management necessary
- we successfully applied our domain stack to the problems in the literature
- our combined strategies were more efficient (fewer iterations) than the current sate of the art